Effect of Flow Resistance through a Narrow Catheterized Stenosed Artery: Analytical Study

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Abstract

In this research, we present an analytical study of the effect of resistance to flow blood through a narrowed artery for catheterization. Vascular diseases are among the most common diseases in countries of the world that can cause cerebral clots or strokes. Where stenosis is the occurrence of abnormal growth on the artery wall, which causes obstruction, in the movement of blood flow, the current model simulates the study of blood flow, given that the blood is Newtonian, incompressible, and steady, with a viscosity and density constant. It was built mathematically to describe the movement of the physiological fluid that represents blood in a gap between two eccentric tubes, where the inner tube is a solid and unified one that represents the moving catheter. While the other is a tapered cylindrical one that represents the artery with overlapping stenosis, where the insertion of the stenosis leads to a change in the blood characteristics (axial velocity, flux and resistance) .The current results showed that the increase in the axial velocity of the stationary catheter is much higher than that of the fixed one, and that the flow resistance increases with the stenosis of the artery.

Keywords: Blood flow, catheterized, stenosed artery, flux, resistance to flow, overlapping.

1. Introduction

One of the leading causes of the deaths in the world is due to heart diseases and the most commonly heard names among the same are ischemia, atherosclerosis, and angina pectoris. Ischemia is the deficiency of the oxygen in a part of the body, usually temporary. It can be due to a constriction (stenosis) or obstruction in the blood vessel supplying the blood in that part [1].

Devajyoti Biwas et al [2], reviewed the pulsatile blood flow through the arterial catheter in the presence of axial stenosis type mild asymmetric with sliding velocity at the narrow wall. The blood was considered to be a Newtonian fluid. It was solved using the turbulence method and the analytical expressions of the velocity. Mekheimer and Kot [3], demonstrated the surgical technique of injecting catheters through arterial stenosis. They illustrated with a mathematical representation the movement of the physiological fluid that represents blood in a gap between two eccentric tubes, where the inner tube is a uniform solid overlapping the moving catheter while the other is an arterial tube representing the artery with intervening stenosis .Mathematically nature of blood by considering it as a Newtonian fluid. The analysis was done for an artery with a mild stenosis. The problem was formulated by using turbulence expansion method to obtain the axial velocity, current function, flow resistance and wall shear. The results showed that there is a significant difference between eccentricity and concentric ring flow through the arterial aerated by catheter. Akbar Zaman et al [4], studied the pulsatile analysis of blood in a catheterized blood vessel. The vessel was designed as a flow of two immiscible fluids. The fluid in the core region is fluid as a non-Newtonian elastic fluid, and the catheter is inside the vessel in the form of a solid tube with a very small radius. Each region numerically by finite difference plot and analytically by Comparison of numerical solution Laplace transforms. Anber Saleem et al [5], discussed the blood flow through the catheter with slight pressure, the stenosis is studied on the wall with a blood clot in the center. The flow in the vessels was considered by the use of the catheter and that the viscous fluid is Newtonian, as well as the characteristic shapes of the stenosis (that is, symmetrical and asymmetric shapes). The results concluded that the speed it increases more for asymmetric stenosis and decreases compared to symmetric stenosis**.** Bakheet and Alnussairy[6], studied the magnetic-mechanical effects of unstable blood flow on Casson's fluid through an artery mean of overlapping stenosis. The mathematical model of the problem was solved by using the pressure correction method with MAC algorithm to understand the phenomenon of blood flow in the diseased artery.

2. Mathematical Formulation

We present the blood symmetric, laminar, steady and incompressible Newtonian fluid of viscosity μ and density ρ constante , this is a three dimensional problem. Let (r, θ, z) the coordinate of a material point in the cylindrical polar coordinate system where the z-axis is taken along the axis of the artery while r, θ are along the radial and circumferential directions, If the eccentricity $\epsilon = 0$, then the catheter is in a concentric position [7]. The catheter is moving in the axial direction with velocity u_0. The geometry of the arterial wall with time-variant overlapping stenosis which is the function $R(z,t)$ define by: [8]

$$R(z,t)=\begin{cases}\left[(mz+R_0)-\frac{\delta\cos\phi}{l_0}(z-d)\left\{\begin{matrix}11-\frac{94}{3l_0}(z-d)+\frac{32}{l_0{}^2}(z-d)^2-\frac{32}{3l_0{}^3}\\(z-d)^3\end{matrix}\right\}\right]\Omega(t), \\ (mz+R_0)\Omega(t) \qquad d\le z\le d+\frac{3l_0}{2}\end{cases} \quad (1)$$

$$\Omega(t)=1-b(cos\omega t-1)exp[-b\omega t], \quad (2)$$

Where, $R(z,t)$ is the radius of the tapered arterial segment in the constricted region, ϕ is the angle of tapering, $3l_0/2$ is the lengh of overlapping stenosis, R_0 is the constant radius of the normal artery without stenosis, d is the location of the stenosis, $\delta\cos\phi$ is taken to be the critical value of the overlapping stenosis, $m=\tan\phi$ represents the slope of the tapered vessel, $b$is a constant, t is the time and ω represents the angular frequency of forced oscillation Separately the boundary of the catheter is described to order ϵ by $r=\sigma+\epsilon\cos\theta$ (obtained by using the cosine rule) where $\epsilon\ll\sigma$ is the parameter which controls the eccentricity of the catheter position, see fig. (1).

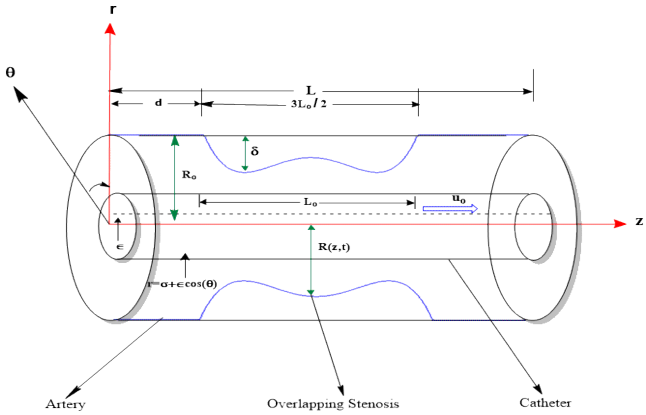

Figure 1: Geometry of Stenosis with Catheter.

The governing equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial r}+\frac{\partial u_z}{\partial z}=0 \quad (3)$$

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)$$

$$=-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 u_r-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial u_\theta}\right) \quad (4)$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$

$$=-\frac{\partial p}{\partial z}+\mu\nabla^2 u_z, \quad (5)$$

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$

$$=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}\right)u_z \quad (6)$$

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}$$

$$= -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right) (7)$$

The boundary condition [3]

$u_r = u_\theta = u_z = 0$ at $r = R(z,t)$ (8)

$u_r = u_\theta = 0, u_z = u_0$ at$r\sigma\epsilon\cos(\theta)$(9)

Where u_r, u_θ, u_z are the velocity components in the $r, \theta$and z directions singly, p is the fluid pressure,

3. Solution of problem

We can solve the exact result to the system of linear equations (3),(4),(7) by using reduce order method as the eccentricity parameter to be very slight ($\epsilon \ll 1$),can be write eq.(7):

$$\frac{\partial u_z^2}{\partial r^2} + \frac{\partial u_z}{\partial r} = \frac{dp}{dz} \qquad (10)$$

We multiply the equation by r

$$r\frac{\partial u_z^2}{\partial r^2} + \frac{\partial u_z}{\partial r} = r\frac{dp}{dz} \qquad (11)$$

On integrating with respect to r , we get

$$\int r\frac{\partial u_z^2}{\partial r^2} + \int \frac{\partial u_z}{\partial r} = \int r\frac{dp}{dz} \qquad (12)$$

$\int r\frac{\partial u_z^2}{\partial r^2}$ by integration by part we get,

$$r\frac{\partial u_z}{\partial r} - u_z + \int \frac{\partial u_z}{\partial r} = \int r\frac{dp}{dz} \qquad (13)$$

$$ru_z' - u_z + u_z = \frac{r^2}{2}\frac{dp}{dz} + A \qquad (14)$$

Integrating with respect to r, yields

$$u_z = \frac{r^2}{4}\frac{dp}{dz} + A\log r + B \qquad (15)$$

New be with eq. (15) the corresponding boundary condition for equation (8), (9) will be in the form we get the axial velocity u_z

$$u_z = \frac{1}{4}\frac{dp}{dz}\left\{\begin{matrix}(r^2 - R^2) \\ + \left[\frac{(R^2 - (\sigma + \epsilon\cos(\theta))^2)}{\log(R/\sigma + \epsilon\cos(\theta))}\right](\log(r/R))\end{matrix}\right\} - u_0\left[\frac{\log(r/R)}{\log(R/\sigma + \epsilon\cos(\theta))}\right] (16)$$

The non-dimensional volumetric flow rate is given by:

$$Q = 2\pi\int_{\sigma+\epsilon\cos(\theta)}^{R} r\,u_z(z,r,t)dr \qquad (17)$$

The axial velocity by eq.(16) is;

$$u_z(z,r) = \frac{1}{4}\frac{dp}{dz}\left\{\begin{matrix}(r^2 - R^2) \\ + \left[\frac{(R^2 - (\sigma + \epsilon\cos(\theta))^2)}{\frac{\log R}{\sigma} + \epsilon\cos(\theta)}\right] \\ (\log r/R)\end{matrix}\right\} - u_0\left[\frac{\log(r/R)}{\log R/\sigma + \epsilon\cos(\theta)}\right] \qquad (18)$$

From equation (17) one now obtains with the help of equation (8) and (9), we obtain as.

$$Q = \frac{-\pi}{8}\left[\frac{dp}{dz}R^4 + \phi(z)\right] \ , \phi(z) = \frac{1}{c(z)} \qquad (19)$$

The pressure drop across the stenosis in the artery of length L is obtained as:

$$\Delta p = \int_0^L \frac{dp}{dz}dz = \frac{8\mu Q}{\pi R^4} + \Psi \qquad (20)$$

Where,

$$\Psi = [\int_0^d |\phi(z)|_{R=1}\, dz \int_d^{d+\frac{3l_0}{2}} |\phi(z)|_{R=1}\, dz + \int_d^L \frac{3}{2}|\phi(z)|_{R=1}dz]\ , (21)$$

4. Results and Discussion

The obtained results were analyzed by the axial velocity u_0 and resistance for different values on a parameter where the highest height, $\delta^*$of the artery stenosis was discussed

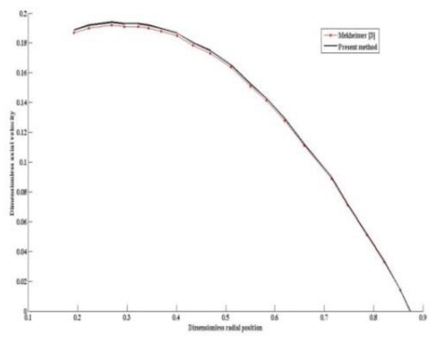

Figure 2: Comparison of axial velocity u_z with Mekheimer et al. (2012) [3] in artery.

In figure 2, we made a comparison of the results of the axial velocity. It is agreement with (Mekheimer et al.,2012 [3]), and very close, from equation of axial velocity u_z in Eq.(16) it is higher for eccentric catheter compared to concentric catheter variation u_z versus r is shown for value at u0=0.1, $\theta = 0^o$, $t = 0.5, z = 1.2, \delta^* = 0.2, \phi = 0, \sigma = 0.1$ and $\epsilon = 0$.

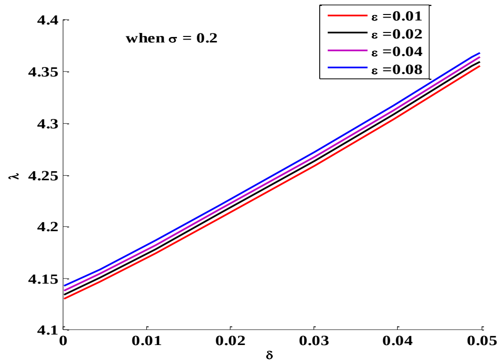

Figure 3 :Variation of resistance impedance λ of different value of ϵ.

In figure 3, we can show of resistanc imedance λ with maximum height of stenosis ,$\delta^*$with defferent value of eccentricity parameter ϵ, when $t = 0.5, \sigma = 0.2$, $u_0 = 0.1, L = 3, \phi = 0$,and $\epsilon = 0.01,0.02,0.04,0.08$

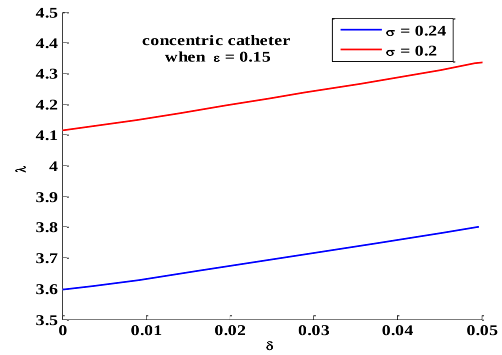

Figure 4: Variation of resistance impedance λ of different value of σ.

While in figure 4, we can show the resistance to flow with the maximum height of stenosis δ^* when the different value of reduce of catheter σ the parameter $\epsilon = 0.15,\ t = 0.5, \sigma = 0.1,\ u_0 = 0.1, L = 3, \phi = 0$,and $\sigma = 0.24, 0.2$

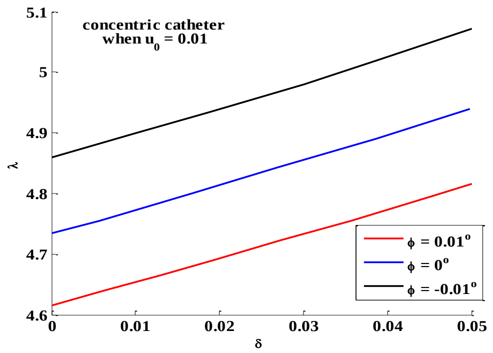

Figure 5: Variation of resistance impedance λ of different value of ϕ

In figure 5, we can show the resistance impedance λ with maximum hight of stenosis $\delta^* = 0.2$ of different value of taper ϕ the parameter $\epsilon = 0.15,\ t = 0.5, \sigma = 0.01,\ u_0 = 0.1, L = 3, \phi = 0.01, 0, -0.01.$

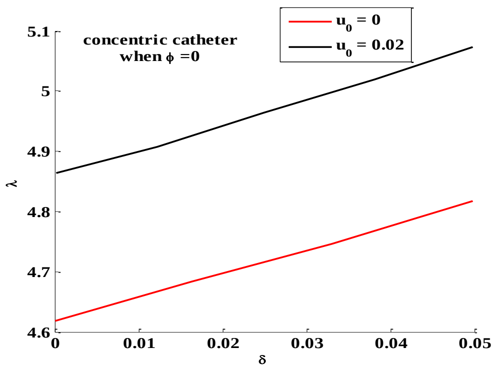

Figure 6: variation of resistance impedance λ with different values of u_0.

In figure 6, we can show resistance impedance with maximum height of stenosis $\delta^* = 0.2$ for different values of velocity of catheter u_0 the parameters $L = 3,\ \epsilon = 0.01, \sigma = 0.1,\ t = 0.5, \phi = 0$ and $u_0 = 0, 0.02$.

5. Conclusion

MATLAB code has been developed to discuss the results that were found in the analysis of the problem. Where the solutions showed that the axial velocity is higher for the centralized catheter than that of the concentric catheter, given that the pointed artery is higher than the non-taper artery, and that the flow resistance increases when the height of the stenosis increases.

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