Mixed Approximate(Hurewicz) Cofibration

Daher Waly Freh - College of Science\ dept. physics\ Wassit University

المستخلص:
في هذا البحث درسنا مفهوما جديدا اسمه اللاتليفات التقريبية - M (المختلطة) واللاتليفات التقريبية هريوز – M (المختلطة)
ظاهرة والي فريح - كلية العلوم /قسم الفيزياء / جامعة واسط

ABSTRAC:
In this papers we study a new concept namely (M-approximate cofibration) Mixed Approximate Cofibration and (M-approximate Hurewicz cofibration) Mixed approximate Hurewicz cofibration.
Most of theorem which are valid for cofibration will also be valid for (M-cofibration); the others will be valid if we add extra conditions . Among the results we obtain are:
1- A product of two Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration) is also a Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration)
- The M-pullback of Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration)is also Mixed approximate Cofibration(Mixed approximate Hurewicz cofibration)
Key words: Mixed approximate (Hurewicz) Cofibration , M-pullback, Lowering homotopy property.
Mathematics Subject Classification 2000 : 46L85, 55P05.
1- Introduction:

In our papers, we introduce and study the new concept of M-approximate Cofibration (M-approximate Hurewicz Cofibration).

We also proof some results and study M-pullback approximate Cofibration and T-lifting function.

Let \( Y \) be any space \( f_1 : X_1 \to Y \), \( f_2 : X_2 \to Y \) are two fiber spaces and \( \alpha : X_2 \to X_1 \) such that \( f_1 \circ \alpha = f_2 \), let \( X = \{X_1, X_2\} \), \( f = \{f_1, f_2\} \) the \( \{X, f, Y, \alpha\} \), has Mixed Lowering homotopy property (M-LHP) w.r.t a space then \( Z \) iff given a map \( h : Y \to Z \) and a homotopy \( g_t : X_1 \to Z \) satisfying \( h \circ f_2 = g_0 \circ \alpha \) then there exist a homotopy \( h_t : Y \to Z \) with \( h_0 = h \) and \( h_t \circ f_1 = g_t \) for all \( t \in I \). M-fiber space is called M-cofibration For class \( \mathcal{R} \) if \( f \) has (M-LHP) for each \( Z \in \mathcal{R} \). The word map in this work means continuous function, \( \mathcal{R} \) means the classes of topological space and \( I \) means \([0,1]\).

2- Preliminaries:

Recalled here same basic concepts and clarify notions used in the sequel

Definition 2-1 [5,6]:

Let \( f, g : E \to B \) be mapping and \( \xi \) be an open cover of \( B \), we say that \( f, g \) are \( \xi \)-closed iff given \( e \in E \) then there exist \( w \in \xi \) such that \( f(e), g(e) \in w \)

Definition 2-2 [4,5,6]:- A map \( p : E \to B \) have to approximate lowering homotopy property (A-LHP) w.r.t \( X \) iff given a map \( h : B \to X \) and a homotopy \( f_t : E \to X \) such that \( h \circ p = f_0 \) and open cover \( \xi \) of \( X \), then there exist a homotopy \( h_t : B \to X \) with \( h_0 = h \) and \( h_t \circ p, f_t \) are \( \xi \)-closed in \( f_t \), for all \( t \in I \). Now let \( \mathcal{R} \) be a given class of topological space, a map \( p \) is a cofibration w.r.t \( \mathcal{R} \) iff \( p : E \to B \) has (LHP) w.r.t each \( X \in \mathcal{R} \)

Definition 2-3 [2,3]:-

1- Let \( X_1, X_2, Y \) be three topological spaces, let \( X = \{X_1, X_2\} \), \( f = \{f_1, f_2\} \) where \( f_1 : X_1 \to Y \), \( f_2 : X_2 \to Y \) are two fiber space and \( \alpha : X_2 \to X_1 \) such that \( f_1 \circ \alpha = f_2 \) then \( \{X, f, Y, \alpha\} \) is a M-fiber space (Mixed fiber space)
If $X_1 = X_2 = X$, $\alpha = \text{identity}$, $f = f_1 = f_2$ then $\{X, f, Y\}$ is the usual fiber space

2- Let $\{X, f, Y, \alpha\}$ be a M-fiber space let $y_0 \in Y$ then $F = \{f(y_0)\}$ is the M-fiber over $y_0$

**Definition 2-4 [2]:** the $\{X, f, Y, \alpha\}$ be a M-fiber structure , $X$ be any space, and $g : Y' \to Y$ be any continuous map into base $Y$

Let $X'_1 = \{(x_1, y') \in X_1 \times Y' : f_1(x_1) = g(y')\}$ and $X'_2 = \{(x_2, y') \in X_2 \times Y' : f_2(x_2) = g(y')\}$ then $X' = \{X'_1, X'_2\}$ is called a M-pullback of $f$ by $g$ and $f' = \{f'_1, f'_2\} : X' \to Y'$ is called induced M-function of $f$ by $g$, that means $f'_1 : X_1 \times Y' \to Y'$, $f'_2 : X_2 \times Y' \to Y'$ are called induced M-function of $\{f'_1, f'_2\}$ by $g$

Define $\alpha' : X'_2 \to X'_1$ by $\alpha'(x_2, y') = (\alpha(x_2), y')$

To show $\alpha'$ is continuous

Since $\alpha' = \alpha \times l_{y'}$, $\alpha$ is continuous and $l_{y'}$ is continuous then $\alpha'$ is continuous

To show $\alpha'$ is commutative

$(f'_1 \circ \alpha')(x_2, y') = f'_1(\alpha'(x_2, y')) = f'_1(\alpha(x_2), y') = y'$, also $f'_2(x_2, y') = y'$ therefore $f'_1 \circ \alpha' = f'_2$
3- M- approximate(Hurewicz) Cofibration

Definition 3-1:- Let \( Y \) be any space \( f_1 : X_1 \rightarrow Y \), \( f_2 : X_2 \rightarrow Y \) are two fiber space and \( \alpha : X_2 \rightarrow X_1 \) such that \( f_1 \circ \alpha = f_2 \), let \( X = \{X_1, X_2\} \), \( f = \{f_1, f_2\} \) the \( \{X, f, Y, \alpha\} \), has Mixed approximate Lowering homotopy property (M-ALHP) w.r.t a space then \( Z \) iff given a map \( h : Y \rightarrow Z \) and a homotopy \( f_t : X_1 \rightarrow Z \) such that \( h \circ f = g \circ \alpha \) and open cover \( \xi \) of \( Z \), then there exist a homotopy \( h_t : Y \rightarrow Z \) with \( h_0 = h \) and \( h_t \circ f_1, f_t \) are \( \xi \)-closed in \( f_t \), for all \( t \in I \).

M-fiber space is called M- approximate cofibration for class \( \mathcal{R} \) if \( f \) has (M-LHP) for each \( Z \in \mathcal{R} \) and the \( \{X, f, Y, \alpha\} \) be a M-fiber structure over \( Y \) , we say that \( \underline{f} \) is M- approximate Hurewicz Cofibration iff \( f \) has (M-ALHP) w.r.t all spaces.

Proposition 3-2:- Every approximate(Hurewicz) Cofibration is Mixed approximate (Hurewicz)Cofibration.
Proof:- let \( \{X, f, Y, \alpha\} \) be a M-fiber space such that \( X_1 = X_2 = X \), \( \alpha = identity \) , \( f = f_1 = f_2 \). let \( h : Y \rightarrow Z \) and a homotopy \( g_t : X_1 \rightarrow Z \) such that \( h \circ f = g \circ \alpha \) and all open cover \( \xi \) of \( Z \) then there exist a homotopy \( h_t : Y \rightarrow Z \) with \( h_0 = h \) and \( h_t \circ f_1, g_t \) are \( \xi \)-closed in \( g_t \) for all \( t \in I \).

Then \( f \) has (M-ALHP) w.r.t \( Z \) , or w.r.t all space
Therefore \( f \) has M- approximate(Hurewicz) cofibration

Proposition 3-3:- let \( \underline{f} : X \rightarrow Y \) and \( \underline{f} : X' \rightarrow Y' \) be two M- approximate Cofibration then \( \underline{f} \times \underline{f} : X \times X' \rightarrow Y \times Y' \) is also M- approximate Cofibration.
Proof:- Let \( Z \) be any arbitrary space
Let \( \underline{h} : Y \times Y' \rightarrow Z \) be map where \( h : Y \rightarrow Z \) and \( h' : Y' \rightarrow Z \) and
Define \( \underline{g} : X \times X' \rightarrow Z \) as \( h^* \circ \underline{f} \rightarrow Z \) and two open covers \( \xi, \xi' \) of \( Z \) such that
\( g_t : X_1 \rightarrow Z \) and \( g_t : X_1 \rightarrow Z \) . since \( \underline{f}, \underline{f} \) are M- approximate Hurewicz Cofibration, then there exist a homotopy \( h_t : Y \rightarrow Z \) with \( h_0 = h \) and \( h_t \circ f_1, g_t \) are \( \xi \)-closed in \( g_t \), and a homotopy \( h'_t : Y' \rightarrow Z \) with \( h'_0 = h' \) and \( h'_t \circ f'_1, g'_t \) are \( \xi' \)-closed in \( g'_t \)
Now for $g_t^*$ and open cover $\xi \times \xi'$ of $Z \times Z$, then there exist $h_t^*: Y \times Y' \to Z$ define as $h_0^* = h^*$ and $h_t^* o (f_1 \times f_1'), g_t^*$ are $\xi \times \xi'$-closed in $g_t^*$. Since $Z$ be any arbitrary
Therefore $f_\times f': X \times X' \to Y \times Y'$ is M-Hurewicz Cofibration

**Proposition 3-4:**- The $M$-pullback of $M$- approximate (Hurewicz) Cofibration is also $M$- approximate (Hurewicz) Cofibration  

**Proof:**- Let $h': Y' \to Z$ and $h: Y \to Z$. Define a homotopy $g_t: X_1 \to Z$ such that $h' o f_2' = g_0 o \alpha$ and open cover $\xi$ of $Z$. Since $f$ has $M$- approximate cofibration then there exist a homotopy $h_t: Y \to Z$ with $h_0 = h$ and $h_t o f_1, g_t$ are $\xi$-closed in $g_t$.

Define $g_t': X_1' \to Z$ such that $h' o f_2' = g_0' o \alpha'$, $g_t' = g_t o L$ and open cover $\xi'$ of $Z$, then there exist a homotopy $h_t': Y' \to Z$ with $h_0' = h'$ and $h_t' o f_1', g_t'$ are $\xi'$-closed in $g_t'$.

Therefore $f': X' \to Y$ has $M$- approximate cofibration
References:


Received ……………………………………………………………………………………………………… (16/2/2010)
Accepted …………………………………………………………………………………………………… (23/6/2010)