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Reduce spherical aberration of doublet of combined quadrupole lenses (rectangular model)

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الخلاصة
حسابات الأمثلية أجريت لإيجاد أفضل الخواص لعدسة مزدوجة رباعية مركبة مكونة من عدسة كهروسكونية ومغناطيسية وذلك من أجل الحصول على عدسة أقل الزيغ كروي.
الأنموذج ألمستطيلي لتوزيع الجهد أستخدم وأيجاد خواص العدسة مثل التكبير ومعامل الزيغ. ولإيجاد أفضل القيم لمعامل الزيغ الكروي تم أخذ تأثيرات تغير كل من عامل التهيج للعدسة والمسافة بين العدستين وطولها الموجي بنظر الاعتبار كعوامل فعالة للوصول للأمثلية.

Abstract
The optimization calculations are made to find the optimum properties of doublet of combined quadrupole lens consist of electrostatic and magnetic lenses to produce reduce spherical aberration lens. The rectangular model is used and finds the properties of lens as the magnification and aberration coefficient. To find the optimum value of spherical aberration coefficient, the effect of both the excitation parameter of the lens, distance between of lenses and the effective wave length of the lens as active parameter to get the optimization.

1. Introduction
The spherical aberration of quadrupole lens when an astigmatics beam impinges on the lens; the field distribution along the lens axis is approximated by a rectangle. These formulas make it possible to derive general expressions for the spherical aberration of a doublet consisting of combined quadrupole lenses [1].

The possibility of reducing the spherical aberration of an electron microscope objective is also rejected. The calculation shows that the random microscopic field due to the discrete nature of the space charge is greater than the small macroscopic field required for correcting the spherical aberration [2].

The correct for spherical aberration usually uses multi-electrode lenses-octypoles and sextupole. It has also been shown that under certain conditions a single three- electrode crossed lens creates a linear image with zero or negative spherical aberration [3].

It is permissible to treat the double lenses used to correct aberration and the double quadrupole lenses used to produce rotating line focus [4].

The chromatic aberrations are corrected by electric and magnetic quadrupole lenses, but quadrupole– octupole correctors are well suited for correcting chromatic and spherical aberrations [5].

The hexapole corrector has the simplest structure yet eliminates only third-order spherical aberration and coma. The quadrupole- octupole corrector eliminates chromatic aberration by means of crossed electric and magnetic quadrupoles and the third- order spherical aberration by octupoles [6].

The optical properties of the new correction lens were calculated by using simulated potential distributions. Aperture aberration coefficients of the new correction lens corrected to less than 0.1mm. It was confirmed experimentally

that the probe size was improved remarkable by excitations of the aperture electrodes [7].

2. Theoretical part

2.1Spherical aberration

The spherical aberrations ΔCD and ΔDC of a qudrupole doublet lens in the plane of the Gaussian image is given by the following formulae [1]

$$\Delta CD = (P_{CD}\, \alpha^3 + S_{CD}\, \alpha\, \gamma^2) \qquad (1)$$

$$\Delta DC = (P_{DC}\, \alpha^2\beta + S_{DC}\, \gamma^3) \qquad (2)$$

Where the subscript pair CD refers to the first lens converges and the second lens diverges; the subscript pair DC. The spherical aberration coefficients, P_{CD}, S_{CD}, P_{DC} and S_{DC} of the doublet lens are determined from (a) the spherical aberration coefficient $C_{30,}$ $C_{12,}$ D_{03} and D_{21} of each of two single lenses forming the doublet and (b) the magnifications M_C and M_D of the first lens [1].

$$P_{CD} = (C_{30})_1 + (D_{03})_2 / M^4_{C1} \qquad (3)$$

$$P_{DC} = (D_{03})_1 + (C_{30})_2 / M^4_{D1} \qquad (4)$$

$$S_{CD} = (C_{12})_1 + (D_{21})_2 / M^2_{C1} M^2_{D1} \qquad (5)$$

$$S_{DC} = (D_{21})_1 + (C_{12)} / M^2_{C1}\, M^2_{D1} \qquad (6)$$

Where the subscript "1" refers to the first lens; and "2" to the second one.

The magnification in both convergence and divergence planes are given by:

$$M_{C1} = 1/\cos\theta_1 - u\beta_1 \sin\theta_1 \qquad (7)$$

$$M_{D1} = 1/\cosh\theta_1 + u\, \beta_1\, \sinh\theta_1 \qquad (8)$$

$M_{C2}=1/\cos\theta_2-u\beta_2\sin\theta_2$ (9)

$M_{D2}=1/\cosh\theta_2+u\,\beta_2\,\sinh\theta_2$ (10)

Where β excitation of the doublet lenses,
u is the object distance and L effective length where θ = βL [8] .

The coefficients of spherical aberration in a rectangular model of the field are given by (1):

$$\frac{(C_{30})_1}{L_1}=\frac{1}{16}[\eta_1^{\ 2}-(1-6\mu_1^{\ 2}+\mu_1^{\ 4})(\frac{\sin 4\theta_1}{4\theta_1})+(\frac{u}{L_1})\xi_1(1-\cos 4\theta_1)+\frac{1}{3}(2-2n_1+3n_1^{\ 2})[3\eta_1^{\ 2}-4(1-\mu_1^{\ 4})$$
$$(\frac{\sin 2\theta_1}{2\theta_1})+(1-6\mu_1^{\ 2}+\mu_1^{\ 4})(\frac{\sin 4\theta_1}{4\theta_1})+4(\frac{u}{L_1})\eta_1(1-\cos 2\theta_1)-(\frac{u}{L_1})\xi_1(1-\cos 4\theta_1)]]$$

$$\frac{(C_{12})_1}{L_1}=\frac{1}{16}[6\eta_1\xi_1+8(\frac{u}{L_1})-[2\xi_1^{\ 2}+(5\xi_1\eta_1-4\mu_1^{\ 2})\cosh 2\theta_1](\frac{\sin 2\theta_1}{2\theta_1})+[2\eta_1^{\ 2}-(\xi_1\eta_1+20\mu_1^{\ 2})\cos 2\theta_1]$$
$$(\frac{\sinh 2\theta_1}{2\theta_1})-2\frac{u}{L_1}(\xi_1\cos 2\theta_1-\eta_1\cosh 2\theta_1)+\frac{u}{L_1}(\eta_1-5\xi_1)\sin 2\theta_1\sinh 2\theta_1-\frac{u}{L_1}(5\eta_1+\xi_1)$$
$$\cos 2\theta_1\cosh 2\theta_1+(2+2n_1-n_1^{\ 2})[-2\eta_1\xi_1-2\frac{u}{L_1}+[2\xi_1^{\ 2}-(\eta_1\xi_1-4\mu_1^{\ 2})\cosh 2\theta_1](\frac{\sin 2\theta_1}{2\theta_1})$$
$$+[2\eta_1^{\ 2}-(\eta_1\xi_1+4\mu_1^{\ 2})\cos 2\theta_1](\frac{\sinh 2\theta_1}{2\theta_1})+2\frac{u}{L_1}(\xi_1\cos 2\theta_1+\eta_1\cosh 2\theta_1)$$
$$+\frac{u}{L_1}(\eta_1-\xi_1)\sin 2\theta_1\sinh 2\theta_1-\frac{u}{L_1}(\eta_1+\xi_1)\cos 2\theta_1\cosh 2\theta_1]]$$

$$\frac{(C_{30})_2}{L_2}=\frac{1}{16}[\eta_2^{\ 2}-(1-6\mu_2^{\ 2}+\mu_2^{\ 4})(\frac{\sin 4\theta_2}{4\theta_2})+(\frac{u}{L_2})\xi_2(1-\cos 4\theta_2)+\frac{1}{3}(2-2n_2+3n_2^{\ 2})[3\eta_2^{\ 2}-4(1-\mu_2^{\ 4})$$
$$(\frac{\sin 2\theta_2}{2\theta_2})+(1-6\mu_2^{\ 2}+\mu_2^{\ 4})(\frac{\sin 4\theta_2}{4\theta_2})+4(\frac{u}{L_2})\eta_2(1-\cos 2\theta_2)-(\frac{u}{L_2})\xi_2(1-\cos 4\theta_2)]]$$

$$\frac{(C_{12})_2}{L_2}=\frac{1}{16}[6\eta_2\xi_2+8(\frac{u}{L_2})-[2\xi_2^{\ 2}+(5\xi_2\eta_2-4\mu_2^{\ 2})\cosh 2\theta_2](\frac{\sin 2\theta_2}{2\theta_2})+[2\eta_2^{\ 2}-(\xi_2\eta_2+20\mu_2^{\ 2})\cos 2\theta_2]$$
$$(\frac{\sinh 2\theta_2}{2\theta_2})-2\frac{u}{L_2}(\xi_2\cos 2\theta_2-\eta_2\cosh 2\theta_2)+\frac{u}{L_2}(\eta_2-5\xi_2)\sin 2\theta_2\sinh 2\theta_2-\frac{u}{L_2}(5\eta_2+\xi_2)$$
$$\cos 2\theta_2\cosh 2\theta_2+(2+2n_2-n_2^{\ 2})[-2\eta_2\xi_2-2\frac{u}{L_2}+[2\xi_2^{\ 2}-(\eta_2\xi_2-4\mu_2^{\ 2})\cosh 2\theta_2](\frac{\sin 2\theta_2}{2\theta_2})$$
$$+[2\eta_2^{\ 2}-(\eta_2\xi_2+4\mu_2^{\ 2})\cos 2\theta_2](\frac{\sinh 2\theta_2}{2\theta_2})+2\frac{u}{L_2}(\xi_2\cos 2\theta_2+\eta_2\cosh 2\theta_2)$$
$$+\frac{u}{L_2}(\eta_2-\xi_2)\sin 2\theta_2\sinh 2\theta_2-\frac{u}{L_2}(\eta_2+\xi_2)\cos 2\theta_2\cosh 2\theta_2]]$$

The coefficients $(C_{30})_1$ and $(C_{12})_1$ refers to the first lens: and the coefficients $(C_{30})_2$ and $(C_{12})_2$ to the second lens.

Where $\mu_1 = \beta_1 u$, $\eta_1 = 1+ \mu_1^2$, and
$\xi_1 = 1- \mu_1^2$.
The coefficients $(D_{03})_1$ and $(D_{21})_1$ are derived from equations (11) and (12) by replacing β_1 and $i\beta_1$.

Where $\mu_2 = \beta_2 u$, $\eta_2 = 1+ \mu_2^2$, and
$\xi_2 = 1- \mu_2^2$.
The coefficients $(D_{03})_2$ and $(D_{21})_2$ are derived from equations (13) and (14) by replacing β_2 and $i\beta_2$.

3. Results and Discussion

The results of the calculation are shown in table (1) gives same values when distance between the lenses where (s = 1 and s =0). This table also gives the value of the spherical aberration coefficient (P_{CD}) is lower than that of the spherical aberration coefficient (P_{DC}) because of equations (3) and (4), we note that P_{CD} and P_{DC} varies inversely with the magnification in both convergence and divergence planes M_{C1} and M_{D1}, because of M_{C1} increases of θ (θ=βL) and M_{D1} decreases of θ. The spherical aberration coefficients (S_{CD} and S_{DC}) is same values for effective lengths of lens ($L_1 = L_2 = 0.7$ mm) and excitations of lens ($\beta_1 = \beta_2 = 0.9$ mm^{-1}).

From table (2) the P_{CD} and P_{DC} coefficients are always positive. The spherical aberration coefficient (P_{CD}) is less than that of the spherical aberration coefficient (P_{DC}). The spherical aberration coefficient (S_{CD}) should equal (S_{CD}).

The results of the optimization calculations for reduce spherical aberration coefficients for combined quadrupole doublet is shown in table (1).

The calculations for spherical aberration coefficients in both table (1) and table (2) are made to show the effect of changing the distance between lenses is equal.

The relations between the spherical aberration coefficients (P_{CD} and P_{DC}) and relative excitation lens and values of effective lengths of the lens are studied, in table (3) the results show that the (P_{CD}) lower than

that of spherical aberration coefficient (P_{DC}).

The same behaviour of spherical aberration coefficients (S_{CD} and S_{DC}).

In table (4) the spherical aberration coefficients decrease with excitation and effective lengths of lenses are increase.

The opposite behaviour for spherical aberration coefficients are found for the case of table (3).

The combined quadrupole doublet the results are shown in tables (3) and (4) appear the relation between reduced spherical aberration coefficients as the excitation of lens (β) and effective length (L) and these coefficients are decrease with increase β and L.

Table (1): The spherical aberration coefficients of the combined quadrupole doublet when s =1 and s = 0.

$L_1 = L_2$	0.7 mm
$\beta_1 = \beta_2$	0.9 mm^{-1}
Object distance (u)	1 mm
$M_{CD} = 1.98$	$M_{DC} = 1.98$
$P_{CD} = -3.53x10^{-4}$	$P_{DC} = -0.638$
$S_{CD} = 0.148$	$S_{DC} = 0.148$

Table (2): The spherical aberration coefficients of the combined quadrupole doublet when s =1 and s = 0.

$L_1 = L_2$	0.8 mm
$\beta_1 = \beta_2$	0.6 mm^{-1}
Object distance (u)	1 mm
$M_{CD} = 1.15$	$M_{DC} = 1.15$
$P_{CD} = 2.5x10^{-3}$	$P_{DC} = 0.074$
$S_{CD} = 0.37$	$S_{DC} = 0.37$

Table (3): The spherical aberration coefficients of the combined quadrupole doublet.

$L_1 = L_2$	1 mm
$\beta_1 = \beta_2$	0.53 mm^{-1}
Object distance (u)	1 mm
$M_{CD} = 1.1$	$M_{DC} = 1.1$
$P_{CD} = 0.044$	$P_{DC} = 1.51$
$S_{CD} = 0.39$	$S_{DC} = 0.39$
Distance between of lenses (s)	1mm

Table (4): The spherical aberration coefficients of the combined quadrupole doublet.

$L_1 = L_2$	1.2 mm
$\beta_1 = \beta_2$	0.63 mm^{-1}
Object distance (u)	1 mm
$M_{CD} = 1.8$	$M_{DC} = 1.8$
$P_{CD} = 6.5X10^{-4}$	$P_{DC} = 0.9$
$S_{CD} = 0.19$	$S_{DC} = 0.19$
Distance between of lenses (s)	1mm

4. Conclusions

From the calculations, it appear that the rectangular model can be used to represent the axial field distribution of the combined quadrupole doublet lenses to find the optimum design of reduce spherical aberration and excitation of the lens (β), the effective length of the lens (L) and distance between of lenses can be used to reduced the values of the spherical aberration coefficient where:

(a) In case table (1) the lower values of the effective length (L) gives the best values of the aberration coefficients while the large values of excitation of the lens (β). The opposite behaviour for spherical aberration coefficients are found for the case in table (2).

(b) In case distance between lenses (s = 1) the larger value in both excitation and effective length for doublet lenses gives the best values of aberration coefficients.

According to the above results, the designer can choose the values of the excitation of the lens and the effective length of the lens to reach a favorable design.

From the calculations one can find the negative values of the aberration coefficients, therefore, the designer can use the combined quadrupole double lenses with these conditions, as a corrector element in the optical system to reduced the total aberration of the system.

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