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# Effect of Scattering Parameter on Small Signal analysis in Direct Modulated Dynamic Semiconductor Lasers

**Mohammed Adnan**
**Lecturer**

**Ayad Zwayen Mohammed**
**Lecturer**

**Razi J. Al-Azawi**
**Lecturer**

**Shams Basil Ali**
**Assistant lecturer**

**Osama Z . Faleh**
**sercher**

**Mudher Mohammed Raoof**
**Chief of Engineer Assistant**

**Jumana Basher Mohammed**
**Programable**

**Israa Basher Mohamme**
**Programable**

**Khansaa Jaafar Abbas**
**Engineer Assistant**

**الخلاصة**

تأثير عنصر التشتت هو ذلك التأثير الذي يؤثر على عمل وخواص ليزرات أشباه الموصلات ذات التضمين المباشر بواسطة تحليل أشارة صغيرة لكثافة تيار الحقن ، حيث يؤثر هذا التأثير على كمية كثافة حاملات الشحنة المكممة المحصورة حيث جزء من هذه الحاملات تفقد في المنطقة الفعالة . عنصر التشتت يقودنا إلى ظهور تأثيرات الربح اللاخطية حيث تقوم بتقليل تردد الاستجابة لعملية التضمين المباشر.
نحن سنستخدم معادلات المعدل الخاصة بليزرات أشباه الموصلات ذات النمط المنفرد والتي تمتلك خواص تستخدم لتضمين إشارة صغيرة ذات موجة مستمرة . نحن نستطيع تمثيل هذا التأثير ومناقشته بواسطة جهاز بسيط متكون من جزئيين.

## Abstract

Effect of scattering parameter is that the effect which it is effected on the operating characteristics of direct modulated semiconductor lasers by small signal analysis for injection current density , where this parameter is effected on amount of quantum confinement carriers density where the part of this carriers density is lost in the active region. The scattering parameter leads to appear the nonlinear gain effects which they are reduced the response frequency of the direct modulation process.

We use the rate equations of the single-mode semiconductor lasers and that they have characteristics are used to continuous-wave small-signal modulation. We can presented this effect and discussion it by the simple two-port device.

## I. Introduction

The main application of semiconductor lasers is as a source for optical communication systems, where the output of the semiconductor lasers is modulated by applying the electrical signal either directly to the optical source or to the external modulator [1]. In direct modulation , the typical approach to enhance modulation band width is to fabricate the high speed semiconductor laser using new materials and structures[1,2]. For example, 33GHz GaAs-based penudomorphic multiple-quantum-well (MQW) ridge-wave guide laser have been reported[2]. However, When the electrical signal applied to semiconductor laser is small signal ,this leads to the distortions that they are appeared in the output of laser will be decreased. But calculation of this signal is depended on the scattering parameters[1,2].

## II. Theory and analysis

For a semiconductor laser which is operating above threshold, the output power depends linearly on the input injection current. It is then expected that for an ac current input with a dc offset above threshold, the optical output power will have a corresponding ac and dc component as depicted in Fig.1[3]

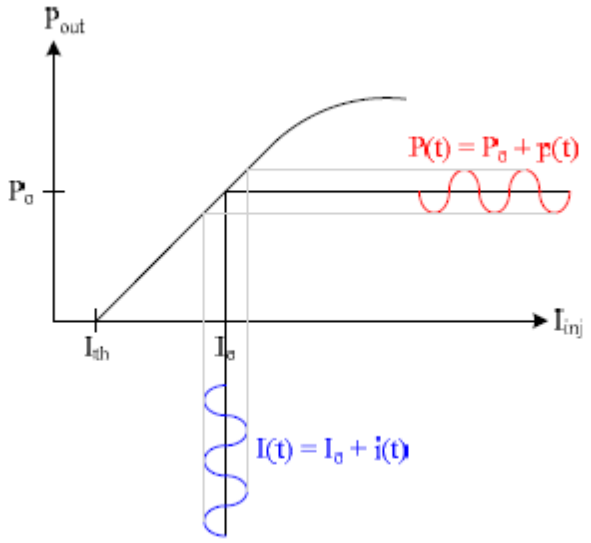


**Fig.1 Graphical representation of output power modulation by input current modulation in a semiconductor laser**

To examine the dynamics of the output power dependence on input current under ac modulation, rate equations for carrier and photon densities are used [3]:

$$\frac{dN}{dt} = \frac{\eta_i J}{qd} - \frac{n}{\tau} - \upsilon g(N) S \quad (1)$$

$$\frac{dS}{dt} = \Gamma \upsilon g(N) S - \frac{S}{\tau_P} + \beta R_{SP} \quad (2)$$

Where N is the carrier density ($cm^{-3}$). $\eta_i$ is the injection efficiency , J is the current density ($A/cm^2$).q is the elementary charge (C ), d is the gain region thickness (cm), $\tau$ is the carrier lifetime (s) , $\upsilon$ is the group velocity of light in the material (cm/s). g(N) is the gain coefficient (1/cm), S is the photon density ($cm^{-3}$) , $\Gamma$ is the optical confinement factor. $\tau_P$ is the photon lifetime (s). $\beta$ is the spontaneous emission factor , and $R_{SP}$ is the spontaneous emission rate per unit volume ($cm^{-3}\ S^{-1}$).

Each term of the rate equations corresponds to a physical process which occurs to change the total number of photons or carries .In eq. ( 1) , the carrier density is increased by the first term , the injected number of carriers per unit volume per second ($\eta_i J/qd$). The carrier density is decreased due to radiative and nonradiative recombination events which are lumped together in the second term($N/\tau$)**,** and also due to stimulated emission ($\upsilon\ g(N)S$).In eq.(2), the photon density is increased by stimulated emission ($\Gamma\upsilon\ g(N)S$)) and a fraction of the spontaneous emission which couples into the lasing mode ($\beta\ R_{SP}$ ). The photon density is decreased due to absorption and cavity losses ($S/\tau_P$ )[3,4].

For this analysis , the gain term will be approximated using a nonlinear model as shown in eq. (3), where $g_0$ and $g'$ are the gain and differential gain , respectively. At the bias point $N_0$ and the $1+\varepsilon S$ term accounts for gain saturation which occurs at high photon densities where $\varepsilon$ is the gain suppression factor[4].

$$g(N) = \frac{g_0 + g'(N - N_0)}{1 + \varepsilon S} \quad (3)$$

The equations for the total injection current density . carrier density , and photon density are given in Equation (4a) , (4b) , and (4c) , respectively[4] :

$$J(t) = J_0 + j(t) \quad (4a)$$

$$N(t) = N_0 + n(t) \quad (4b)$$

$$S(t) = S_0 + s(t) \quad (4c)$$

For small-signal analysis , the ac variations j(t). n(t) .and s(t) are assumed to be very small compared to their dc counterparts $J_0$ , $N_0$, and $S_0$ . The dc steady state solutions to the rate eqs. (1) and (2) can be obtained by setting the derivatives as well as the ac components of the current , carrier , and photon densities all equal to zero ,and solving for $S_0$ and $N_0$ in terms of $J_0$ [5].

Assuming an ac sinusoidally varying current , carrier , and photon density, the rate equations can be analytically solved through the use of the phasor

expressions j(ω),n(ω),and s(ω), where ω is the angular frequency of modulation . Substituting in the steady state solutions of $S_0$ and $N_0$ into the solution for s(ω) and dividing by j(ω) yields the small-signal modulation response[5] :

$$|M(\omega)|^2 = \left|\frac{s(\omega)}{j(\omega)}\right|^2 = \left(\frac{\Gamma\tau_P}{qd}\right)^2 \frac{\omega_r^4}{(\omega^2-\omega_r^2)^2+\omega^2\gamma^2} \tag{5}$$

Where ( assuming negligible spontaneous emission ) the relaxation oscillation frequency $f_r=\omega/2\pi$ and the damping coefficient γ are given by Equations (6) and (7), respectively.

$$f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi}\sqrt{vg'\frac{S_0}{\tau_P}} \tag{6}$$

$$\gamma \approx vg'S_0\left(1+\frac{\varepsilon}{vg'\tau_P}\right)+\frac{1}{\tau} \tag{7a}$$

$$= Kf_r^2+\frac{1}{\tau} \tag{7b}$$

In eq. (7b) , the K factor ( with units of seconds ) has been defined for convenience .

The small signal bandwidth of the device is defined as the frequency at which the modulation bandwidth is equal to one-half the value at dc . this corresponds a 3-dB drop on a logarithmic scale . and as such , the bandwidth is often termed the 3-db frequency . it can be shown that the maximum 3-dB bandwidth possible occurs when the following condition is met[4] :

$$2\omega_r^2 = \gamma^2 = K^2 f_r^4 \tag{8}$$

From eq. (8) , the theoretical maximum relaxation oscillation freque found to be

$$f_{r\max} = \frac{2\pi\sqrt{2}}{K} \tag{9}$$

Unfortunately, this model is somewhat poor for a real semiconductor laser. Parasitic effects such as series resistance, capacitance between the contact pads, and capacitance due to the oxide region create the need for a more accurate model oxide-confined devices are primarily limited due to the capacitance of the oxide region. Adding another term to the modulation transfer function yields the three pole model[5,6]:

$$|M(\omega)|^2 = \frac{1}{1+(\omega/\omega_{par})^2}\left|\frac{s(\omega)}{j(\omega)}\right|^2 = \left(\frac{\Gamma\tau_P}{qd}\right)^2 \frac{1}{1+(\omega/\omega_{par})^2}\frac{\omega_r^4}{(\omega^2-\omega_r^2)^2+\omega^2\gamma^2} \tag{10}$$

Where $\omega_{par}$ is the frequency of the pole due to the parasitics.

To accurately calculate the small-signal modulation characteristics of the photonic crystal semiconductor, scattering parameters were employed. Fig.2 illustrates a simple two-port device (due to high frequency operation, transmission lines must be considered) [6].

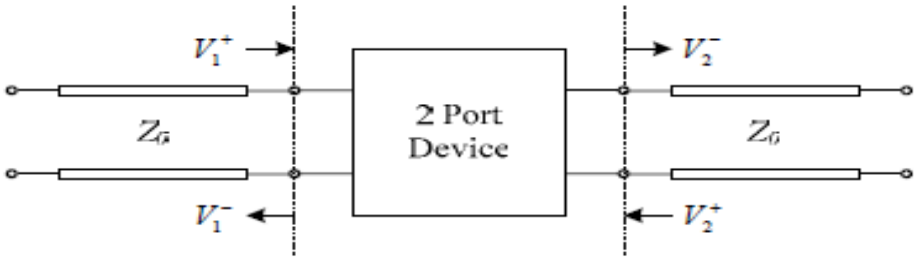


**Fig.(2) Linear two – port device model**

The voltage and current at port one and port two are defined as

$$V_1^{total} = V_1^+ + V_1^- and V_2^{total} = V_2^+ + V_2^- \quad (11)$$

$$I_1^{total} = \frac{V_1^+ - V_1^-}{Z_0} and I_2^{total} = \frac{V_2^+ - V_2^-}{Z_0}$$

The normalized forward and backward traveling waves are defined as

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} and b_1 = \frac{V_1^-}{\sqrt{Z_0}} \quad (12)$$

$$a_2 = \frac{V_2^+}{\sqrt{Z_0}} and b_2 = \frac{V_2^-}{\sqrt{Z_0}}$$

The system scattering parameters ( or S- parameters ) are defined as

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2 \quad (13)$$

By matching the end of transmission line connected to port 2 (in Fig. (2)), the reflected wave $b_2$ becomes zero . it is straightforward to show that $S_{21}$ (often called the system transmission ) is given by[6]

$$S_{21} = \frac{V_2}{V_1}(1 + S_{11}) \quad (14)$$

And for small or constant $S_{11}$ , which is valid when the two port device is either matched to $Z_0$ or has a very small reactance associated with it , the transmission $S_{21}$ is proportional to the voltage gain of the system[7]

$$S_{21} \propto \frac{V_2}{V_1} \quad (15)$$

Under bias , all semiconductor lasers show a small- signal linear relationship between injection current and input voltage . therefore , by using a high-speed photodetector which gives a small-signal linear voltage response which is

proportional to the input light power, the transmission $S_{21}$ is proportional to the small signal modulation response[6,7]

$$|S_{21}(\omega)| \propto |M(\omega)| = \left|\frac{s(\omega)}{j(\omega)}\right| \quad (16)$$

i.e. The transmission $S_{21}$ is calculate as a function of frequency. Therefore $S_{21}$ is given as

$$S_{21}(\omega) = (1 - \Gamma^{(\text{port one})}) \quad \sqrt{\frac{\epsilon_0}{\mu_0}}|M(\omega)| \quad (17)$$

Where $\epsilon_0$ and $\mu_0$ are dielectric and permeability in the vacuum.

Table(1) is parameters values of the 1.3μm Buried hetero structure lasers[2].

**Table(1)[2]**
**The parameters values of 1.3μm Buried hetero-structure laser**

| Parameter | Symbol | Value |
|---|---|---|
| Active-layer thickness | D | 0.2 μm |
| Cavity length | L | 250 μm |
| Active-region width | W | 2 μm |
| Group velocity | $c/\mu_g$ | m/s |
| Effective refractive index | $\overline{\mu}$ | 3.4 |
| Group refractive index | $\mu_g$ | 5 |
| Carriers recombination lifetime | $\tau_{SP}$ | 2.2ns |
| Photon lifetime | $\tau_P$ | 1.6ps |
| Confinement factor | Γ | 0.3 |
| Threshold current | $I_{th}$ | 15.8mA |
| Fraction of spontaneous emission entering the lasing mode | β | $10^7$ $s^{-1}$ |
| Gain compression factor | ε | $3\times10^{-12}$ $m^2/s$ |
| Confinement factor of port one | $\Gamma^{(\text{port one})}$ | 0.023 |

## III. Results and Discussions

From the table (1) and the eqs.(10) and (17),Fig.3 shows the relaxation resonant frequency ($f_r$) as a function of the square root of the photon density ($\sqrt{P}$) , we can observed from this figure the relaxation resonant frequency without scattering parameter effect is higher than these frequency with scattering parameter effect due to the scattering parameter on the small signal will decrease the injection carriers density. In addition, this different between two frequencies will lead to nonlinear gain effects where this effect will reduce the response frequency due to the K- factor will decrease{ where we can calculated the K- factor from Fig.3 which it is represented the slope of the curve } ,this leads to the damping rate and the response frequency will reduce .

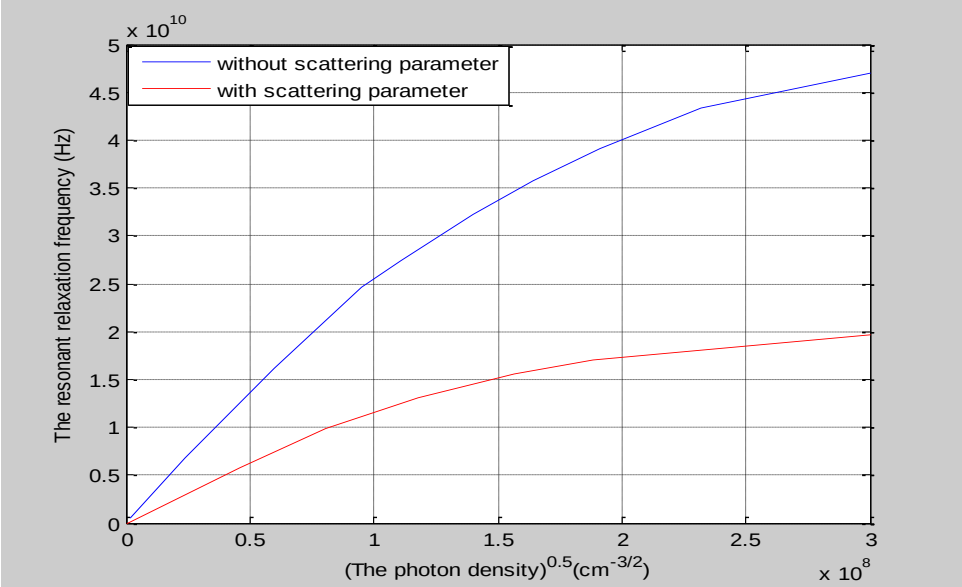


Fig.3 The resonant relaxation frequency with respect to (the photon density)$^{0.5}$

Fig.4 shows frequency response as a function of the modulation frequency where we are observed the different between two cases of frequencies response s due to the effect of the scattering parameter on the small signal

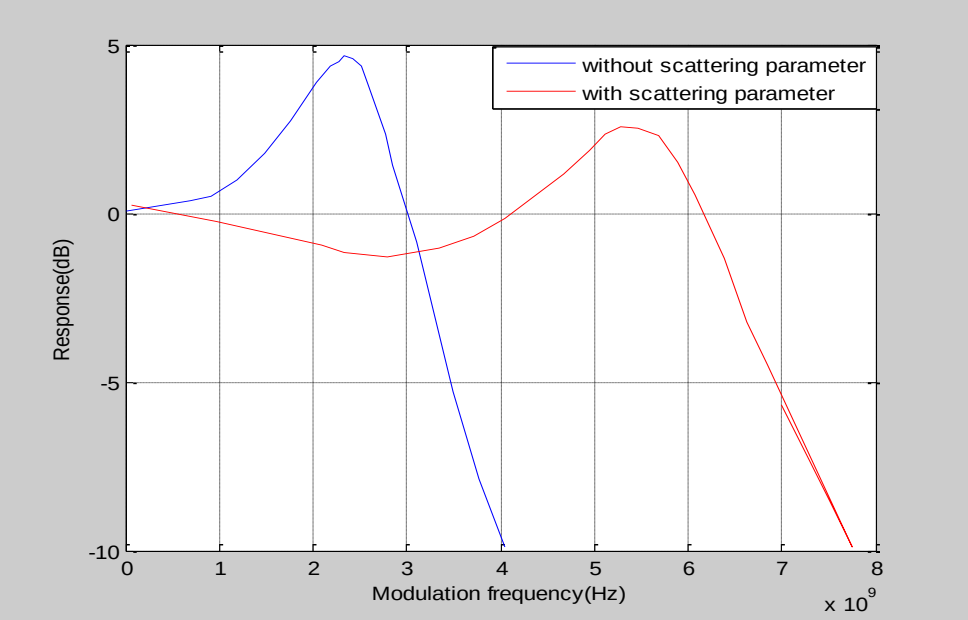


**Fig.4 The Response a function of the Modulation frequency**

## V. Conclusions

The single mode semiconductor lasers were characterized under small signal modulation. The modulation response is limited due to the scattering parameter that this is appeared in the small signal analysis where this is effected on amount of the quantum confinement carrier density where the part of this carriers will loss in the active region . In addition , this effect leads to the appearance the nonlinear gain effects, where this effects will reduce the damping rate and the response frequency. This means the performance of the modulation for semiconductor lasers will limit.

## References


**[1]** S. L. Chuang, Physics of Optoelectronic Devices. New York: Wiley Interscience, 1995.

**[2]** L. A. Coldren and S. W. Corzine, Diode Lasers and Photonic Integrated Circuits.New York: Wiley Interscience, 1995.

**[3]** Agilent, Application Note AN 154, pp. 3-6, 2000.

**[4]** T. R. Chen, B. Zhao, L. Eng, Y. H. Zhuang, J. O'brien, and A. Yariv, "Very high modulation efficiency of ultralow threshold current single quantum well InGaAs lasers," Electronics Letters, vol. 29, pp. 1525-1526, 2006.

**[5]** C. Carlsson, H. Martinsson, R. Schatz, J. Halonen, and A. Larsson, "Analog modulation properties of oxide confined VCSELs at microwave frequencies," Journal of Lightwave Technology, vol. 20, no. 9, pp. 1740-1749, 2002.

**[6]** A. N. Al-Omari and K.L. Lear, "Polyimide-planarized vertical-cavity surfaceemitting lasers with 17.0-GHz bandwidth," IEEE Photonics Technology Letters, vol. 16, no. 4, pp. 969-971, 2004.

**[7]** E. W. Young, "A comprehensive study of hybrid vertical-cavity surface-emitting lasers," Ph.D. dissertation, University of Illinois at Urbana-Champaign, 2002.



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